PLANAR METRIC RECTIFICATION BY ALGEBRAICALLY ESTIMATING THE IMAGE OF THE ABSOLUTE CONIC

Yisong Chen and Horace H S Ip
Image Computing Group, Department of Computer Science,
AIM*tech Center, City University of Hong Kong*

ABSTRACT

A new metric rectification method for planar homography is proposed based on a closed form algebraic solution of the image of the absolute conic on the image plane. Our solution allows shape measurement to be made directly on the image plane without explicitly computing the homography matrix or recoreringing the rectified image. We show that the invariance property of the relationship between the circular points and the absolute conic under projective transformation can effectively do planar metric rectification. In this approach, the image of the absolute conic is solved algebraically to achieve metric rectification based only on the vanishing line and the image of one arbitrary circle on the world plane extracted automatically from the image plane. The process of conic solving introduces no errors and the performance of the method is mainly dependent on the robustness of the straight line and ellipse fitting processes. The fitting scheme suggested in the paper is robust and give good results in most cases.

Keywords: planar homography, metric rectification, ellipse fitting, absolute conic, circular points.

1. INTRODUCTION

Metric rectification aims to remove the projective distortion in the perspective image of a world plane to the extent that similarity properties on the original plane could be measured [1]. It is well known that a planar homography can be determined uniquely from four or more point correspondences. In the cases where point correspondences are not readily available the solution can be acquired through either stratified or unstratified rectification [2]. Both approaches are based on the principle that the metric properties can be recovered once the absolute conic is identified on the image plane.

In this paper, a new method is proposed to estimate the image of the absolute conic (IAC) algebraically. The mathematical foundation of our method is the invariance property of the relationship between the absolute conic and the circular points under projective transform. In our approach, the algebraic representation of the vanishing line and the image of an arbitrary circle on the world plane are first identified on the image plane. Then the images of two circular points are computed on the image plane by algebraically solving the intersection of the image of the identified circle and the vanishing line. Finally, the image of the absolute conic is calculated directly from two identified perspectively transformed circular points.

The rest of the paper is organized as follows. Section 2 offers a brief overview of the related work. Section 3 highlights the principle for computing the absolute conic algebraically given the vanishing line and the image of one circle under perspective transformation. Section 4 explains the implementation details of feature extraction and ellipse fitting. Section 5 gives the experimental results. Finally, Section 6 concludes the paper.

2. RELATED WORK

A 2D homography denotes a planar projective transformation represented by a non-singular 3*3 matrix H. Points on the image plane, x', are related to points on the world plane, x, as x'~Hx, where x and x' are homogeneous 3-vectors, and the notation "~" indicates equality up to a nonzero scale factor [6]. Metric properties on the world plane can be used to partially determine the projective transformation up to a particular ambiguity [3][4]. This partial determination requires far less information about the world plane to be known, but is nevertheless sufficient to enable metric measurements of entities on the world plane to be made from their images [7][9].

The concept of strata structure of projective, affine and metric representations was first described by Faugeras *et al.*[5]. Stratified rectification stems from this concept. In this approach, an affine transformation is first determined through identifying the vanishing line on the image plane, which is the image of the line at infinity on the world plane. Then the affinity can be reduced to similarity by additional information, such as a length ratio or a known angle [1][2][6].

Metric rectification can also be achieved via unstratified method [11]. Let ω be the conic dual of the circular points. It can be shown that orthogonal lines are conjugate with respect to ω under perspective transformations. Each pair of orthogonal lines thus places a linear constraint on ω. Consequently, five right angles are sufficient to determine ω linearly, provided that lines of more than two orientations are included. Once the absolute conic is identified on the image plane, projective distortion may be rectified up to a similarity [11][12]. However, right-angle based approach requires 5 orthogonal line pairs in more than two orientations to be identified on the image plane. This is quite a strong constraint in practice.

This paper suggests an alternative unstratified method for identifying the absolute conic on the image plane. We show that the relationship between the absolute conic and the circular points is invariant under 2D homography and that metric rectification can be achieved by identifying the images of the circular points via algebraically solving the intersection of the image of one circle on the world plane and the vanishing line. The accuracy of the approach depends on the precision of the vanishing line and the transformed circle on the image plane.

The major contribution of the paper is an algebraic framework to solve the absolute conic on the image plane given the vanishing line and one circle under the projective transformation. An implementation scheme is also addressed to make the framework practical.

3. SOLVING THE IMAGE OF THE ABSOLUTE CONIC

It is a well known result in projective geometry that every circle on the plane intersects the line at infinity at two fixed complex circular points $I=(1,i,0)^T$ and $J=(1,-i,0)^T$. These two points are fixed under any similarity transformation. It has been shown that identifying the circular points (or equivalently their dual) allows the recovery of similarity properties. The dual conic to the circular points, ω, is called the absolute conic of the plane, which is also fixed under similarity transformations. ω can be associated with the circular points through the following equation [6]:

$$\omega = IJ^T + JI^T \tag{1}$$

Suppose that the homography is represented by a 3*3 matrix H which transforms the point x on the world plane to the point x' on the image plane by P(x)=x'~Hx. It is easily shown that one line conic C on the world plane is mapped to the conic C' on the image plane by $C'=HCH^T$. Accordingly, the image of the absolute conic can be computed with the following equation:

$$\begin{aligned}\omega' &= H\omega H^T = H(IJ^T + JI^T)H^T \\ &= HI(HJ)^T + HJ(HI)^T = I'J'^T + J'I'^T\end{aligned} \tag{2}$$

That is, equation (1) still holds on the perspective image plane, as long as the circular points and the absolute conic are replaced by their corresponding images under the homography.

Equation (2) shows that ω' can be computed once I' and J' are identified. Note that I and J are the intersection of the line at infinity and any circle on the world plane, which is invariant under arbitrary 2D perspective transformation. As a result, I' and J' can be identified on the image plane by solving the intersection of the vanishing line, l', and the imaged circle, C', which is in general an ellipse on the image plane.

Under homogeneous representation, C' is a conic on the image plane with the form of a symmetric matrix as in equation (3), and l' is a line on the image plane with the form of a column vector as in equation (4).

$$C' = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \tag{3}$$

$$l' = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^T \tag{4}$$

The intersection of C' and l' is the solution of the following equation group:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \tag{5}$$

$$l_1 x + l_2 y + l_3 = 0 \tag{6}$$

As long as l is not kept infinite on the image plane, it can never occur that both l_1 and l_2 are zeros. Without loss of generality, suppose that l_2 is nonzero and equation (6) can be written as

$$y = u_1 x + u_2 \tag{7}$$

where $u_1 = -l_1 / l_2, u_2 = -l_3 / l_2$.

Substitute (7) into (5), we get quadratic equation (8) about x.

Since all coefficients in equation (8) are real numbers, we can easily solve Equation (8) to obtain a pair of conjugate roots $\mathrm{Re}(x_0) \pm i \cdot \mathrm{Im}(x_0)$. Similarly, the corresponding solutions of y are also conjugate values $\mathrm{Re}(y_0) \pm i \cdot \mathrm{Im}(y_0)$. Therefore, the homogenous coordinates of the two solutions of the problem can be written as equation (9).

Substitute (9) into (2), after simplification we get the solution for ω' as in equation (10).

It is worth noting that ω' is a real symmetric matrix although I' and J' are complex. With equation (10) the angle θ between two lines can be calculated with the following equation (11):

$$\cos(\theta) = \frac{l'^T \omega' m'}{\sqrt{(l'^T \omega' l')(m'^T \omega' m')}} \tag{11}$$

Equation (11) is invariant to projective transformations. Therefore, it can be used to compute any angle between two lines on the world plane directly from the image plane. This means that metric rectification is achieved.

4. PRACTICAL APPLICATIONS

To apply the framework of Section 3 in practice, first some edge features should be extracted from the image plane. Then the work of identifying vanishing line and ellipse follows. In this section, we suggest a convenient and robust approach to handle these problems.

4.1. Vanishing line identifying

Affine properties may be recovered by specifying the vanishing line on the image plane. The problem of identifying the vanishing line is not the focus of this paper. Here we only briefly outline one generic approach. The vanishing line can be identified from the image plane with the help of several sets of parallel lines [4]. Each line can be extracted by least squares fitting of several feature points on the line. Each parallel line set determines one vanishing point and all vanishing points jointly determine the vanishing line.

4.2. Ellipse extraction

Roughly speaking, a conventional edge detector reports much more responses than what are really needed. To guarantee a reliable fitting, all false alarms should be removed before further work can be done. These can be realized with the help of color information. Firstly, a conventional edge-detecting algorithm is executed to give the initial edge responses. Secondly, each of these responses is checked to see whether its local color distribution is consistent with the given information of the edge of the target circle. Finally, some human intervention may be introduced if necessary to guarantee no false alarms survive. The approach is robust enough to exclude all outliers in most practical situations.

$$(a + bu_1 + cu_1^2)x^2 + (bu_2 + 2cu_1u_2 + d + eu_1)x + (cu_2^2 + eu_2 + f) = 0 \tag{8}$$

$$\begin{aligned} I' &= (x_0, y_0, 1) = (\mathrm{Re}(x_0) + i \cdot \mathrm{Im}(x_0), \mathrm{Re}(y_0) + i \cdot \mathrm{Im}(y_0), 1) \\ J' &= (\bar{x}_0, \bar{y}_0, 1) = (\mathrm{Re}(x_0) - i \cdot \mathrm{Im}(x_0), \mathrm{Re}(y_0) - i \cdot \mathrm{Im}(y_0), 1) \end{aligned} \tag{9}$$

$$\omega' \sim I'J'^T + J'I'^T \sim \begin{bmatrix} \mathrm{Re}(x_0)^2 + \mathrm{Im}(x_0)^2 & \mathrm{Re}(x_0)\mathrm{Re}(y_0) + \mathrm{Im}(x_0)\mathrm{Im}(y_0) & \mathrm{Re}(x_0) \\ \mathrm{Re}(x_0)\mathrm{Re}(y_0) + \mathrm{Im}(x_0)\mathrm{Im}(y_0) & \mathrm{Re}(y_0)^2 + \mathrm{Im}(y_0)^2 & \mathrm{Re}(y_0) \\ \mathrm{Re}(x_0) & \mathrm{Re}(y_0) & 1 \end{bmatrix} \tag{10}$$

4.3 Ellipse fitting

Ellipse identifying is the major challenge in applying our algorithm in practice. All ellipse edge points have to be fitted to obtain the algebraic conic representation of the ellipse in the image plane coordinate. The fitting problem is to solve the vector v=[a,b,c,d,e,f] in equation (5) subject to the constraint $b^2 - 4ac < 0$. It has been treated with different approaches such as curve fitting and Hough transform [8]. We adopt a practical approach that performs a general conic fitting by least-squares method and then reject non-elliptical fits. Details are as follows.

Given all responses reported by the ellipse detecting algorithm the problem is that of solving a set of over-determined equations of the form Av=0, where A is the coefficient matrix determined by the coordinates of all edge points of the ellipse. The trivial solution v=0 is not of our interest. Observe that v is to be solved up to a nonzero scale factor. So a reasonable constraint would be to seek a solution for which ||v||=1. In general, such a set of equations will not have an exact solution. We will normally seek a least-squares solution instead. The problem may be stated as follow:

Find the v that minimizes ||Av|| subject to ||v||=1.

The problem can be settled by performing an SVD decomposition, $A=UDV^T$, of the coefficient matrix A. The solution v is exactly the last column of V [6]. Data normalization is an essential step in the algorithm for a better precision [10].

The constraint $b^2 - 4ac < 0$ is not considered in the above approach. Nevertheless, experiments show that with a carefully designed ellipse extraction routine the constraint always holds.

Once the work of ellipse fitting is completed, the method described in Section 3 can be used to find the imaged circular points and estimate the image of the absolute conic. Since the process of solving IAC is totally algebraic and thus introduces no errors, the performance of the algorithm is mainly determined by the accuracy of extracting the vanishing line and ellipse in practice.

5. EXPERIMENTAL RESULTS

The first scene in our experiments is a photograph of two signs on the mosaic wall, as shown in Figure 1-a. Our target is to estimate the absolute conic on the image plane. Since the mosaic is not square, right-angle based approach is not applicable here. The result of edge feature extraction is shown in Figure 1-b. It is evident from Figure 1-b that SUSAN edge detector is able to output enough valid edge point features on both the mosaic lines and the ellipses associated with the sign.

The vanishing line is estimated as described in Section 4.1, and one target feature ellipse is identified as described in Section 4.2-4.3. After that, the intersection of the ellipse and the vanishing line is solved algebraically as described in section 3 to estimate the image of the absolute conic.

Several feature points are flagged on the image plane as shown in Figure 1-a to assess the performance. Some representative angles formed by these points in the image are selected and Equation (11) is used to calculate these angles with the estimated absolute conics. Table 1 gives cosine values and angle values (in degree) of the acute angles between several lines on the world plane. We can see that all different angles can be estimated quite precisely. Orthogonal lines always result in cosine values close to 0 and parallel lines always lead to cosine values close to 1. Three inner angles of the regular triangle are all very close to 60 degrees. The scale of the width and the height of the mosaic on the wall is estimated to be 0.4982, which is very close to the actual measured value of 0.50 (The measured width is 12.5cm and height is 25.0cm).

The second scene is a photograph of the ground of a plaza with one circle and several star-like stripes, as shown in Figure 2-a. Again right-angle based approach is not applicable here for the lack of sufficient right angles. Although this scene is much noisier compared with the previous one, the distinct magenta color of the circle and the stripes helps the edge detector find sufficient features of both ellipse edges and stripe edges. We use the two edges of each stripe as parallel line pairs and the edges of the inner ring as the target ellipses. The ellipse detection result is shown in Figure 2-b. Both edges of the inner ring are extracted with high accuracy.

The circular points and the absolute conic can be computed with either of the two edges of the inner ring. We give both results in Table 2. Both absolute conics do well in metric rectification. The results in table 3 successfully verified that each neighboring stripe pairs form a 30 degree angle. In addition, further planar measurement can be done easily once the absolute conic is estimated from the image plane. For instance, if the width of one stripe is known to be 45cm, the diameter of the outer circle can be estimated to be about 1000cm by using the Sine theorem on triangle ABC in Figure 2-a. The value is almost the same as the actual measurement.

In these two experiments, the largest deviation from the expected value is only 1.9 degrees, which is within the accepted range considering the image noise and the feature extraction errors. This shows that the approach addressed in this paper is an effective and practical one for metric rectification.

6. CONCLUSION

This paper shows that the image of the absolute conic can be estimated algebraically given the vanishing line and the image of one arbitrary circle on the image plane. Our experiments show that the estimated absolute conic works well to estimate metric properties directly through the image plane without explicitly computing the homography parameters or the rectified image. Future work includes more robust feature extraction algorithm, more reliable fitting algorithm and generalization to 3D environments.

ACKNOWLEDGEMENT

The work described in this paper was supported by a grant from CityU (Project #9010005) and HKSAR RGC CityU1150/01E.

REFERENCES

[1] Criminisi, A., Reid, I. and Zisserman, A., Single View Metrology, in International Journal of Computer Vision, vol. 40, no. 2, pages 123-148, November, 2000.

[2] Liebowitz, D. and Zisserman, A., Metric Rectification for Perspective Images of Planes, In Proc. CVPR, pages 482-488, June, 1998.

[3] Caprile B, Torre V, Using vanishing points for camera calibration, Int J Comput Vision, Vol. 4(2), pp. 127–140, 1990.

[4] E. Guillou et al. Using vanishing points for camera

calibration and coarse 3D reconstruction from a single image, The Visual Computer, 16(7), 2000, pp. 396-410.
[5] Olivier Faugeras, Stratification of 3-D vision: projective, affine, and metric representations, Journal of the Optical Society of America A, 12(3):465-484, March 1995.
[6] R. Hartley, A. Zisserman, Multiple view geometry in computer vision, Cambridge University, 2000.
[7] Zhengyou Zhang: Flexible Camera Calibration by Viewing a Plane from Unknown Orientations. ICCV 1999: 666-673.
[8] Fitzgibbon, M. Pilu , R.Fisher `Direct least-square fitting of Ellipses , IEEE Trans. PAMI, June 1999.
[9] S. M. Smith and J. M. Brady, SUSAN - a new approach to low-level image processing, International Journal of Computer Vision, 23(1):45--78, 1997.
[10] R. Hartley. In defence of the 8-point algorithm, In Proc. 5th ICCV, pages 1064–1070, Boston, MA, June 1995.
[11] B. Triggs, Autocalibration from Planar Scenes, ECCV, pp. 89–105, 1998.
[12] P. F. Sturm and S.J. Maybank, On Plane-Based Camera Calibration:A General Algorithm, Singularities, Applications, CVPR, pp. 432–437, 1999.

Table 1. Results of mosaic wall scene experiment

	AD/AB	AD/BC	BC/BD	CD/BD	EF/FG	FG/GE	GE/EF	FG/AD
Cos(θ)	0.0165	0.9999	0.4300	0.8951	0.5027	0.4706	0.5262	0.9999
θ	89.055	0.120	64.530	26.480	59.822	61.926	58.252	0.729

Table 2. Estimated circular points and absolute conic parameters in plaza scene image

	X	Y	a	b/2	c	d/2	e/2	f
Ellipse1	$141 \pm 851i$	$721 \pm 343i$	$7.44*10^5$	$3.94*10^5$	$6.38*10^5$	$1.41*10^2$	$7.21*10^2$	1.00
Ellipse2	$128 \pm 822i$	$716 \pm 332i$	$6.93*10^5$	$3.64*10^5$	$6.23*10^5$	$1.28*10^2$	$7.16*10^2$	1.00

Table 3. Results of the plaza scene experiment

	L1/L2	L1/L3	L1/L5	L1/L7	L3/L5	L3/L7	L5/L7
Innerellipse Cos(θ)	0.9999	0.8587	0.4891	0.0040	0.8660	0.5142	0.8742
Innerellipse θ (degree)	0.046	30.715	60.719	89.77	30.004	59.055	29.051
Outerellipse Cos(θ)	0.9999	0.8602	0.4756	0.0266	0.8577	0.4860	0.8662
Outerellipse θ (degree)	0.045	30.664	61.602	88.475	30.938	60.922	29.985

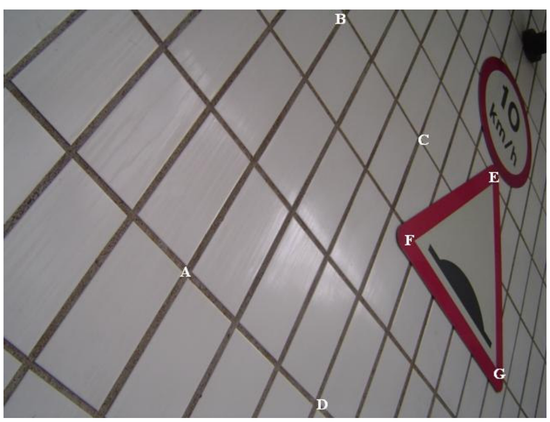

Figure 1-a. A mosaic wall with two signs


Figure 1-b. The edge feature extraction result

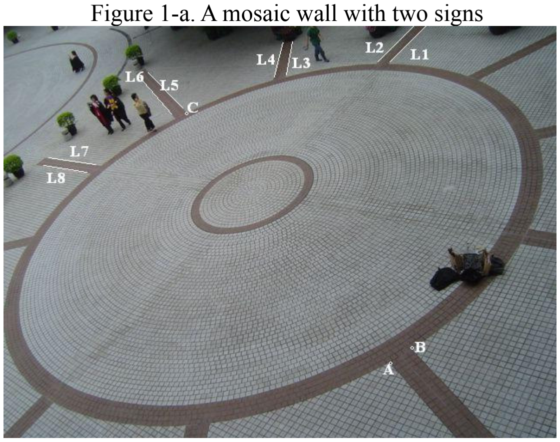

Figure 2-a. The photograph of one plaza

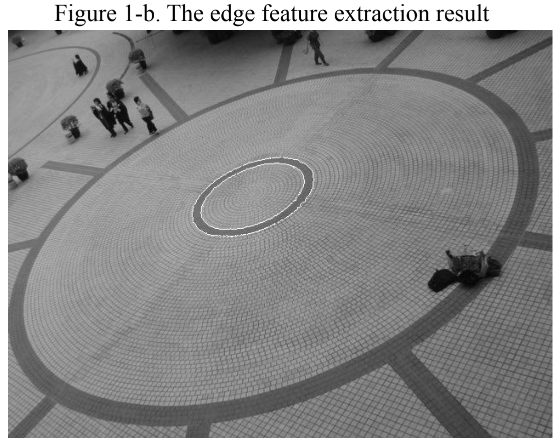
Figure 2-b. two ellipses extracted from the photograph