Image and Vision Computing Single view vision

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Projective geometry

How does a scene map to its image?

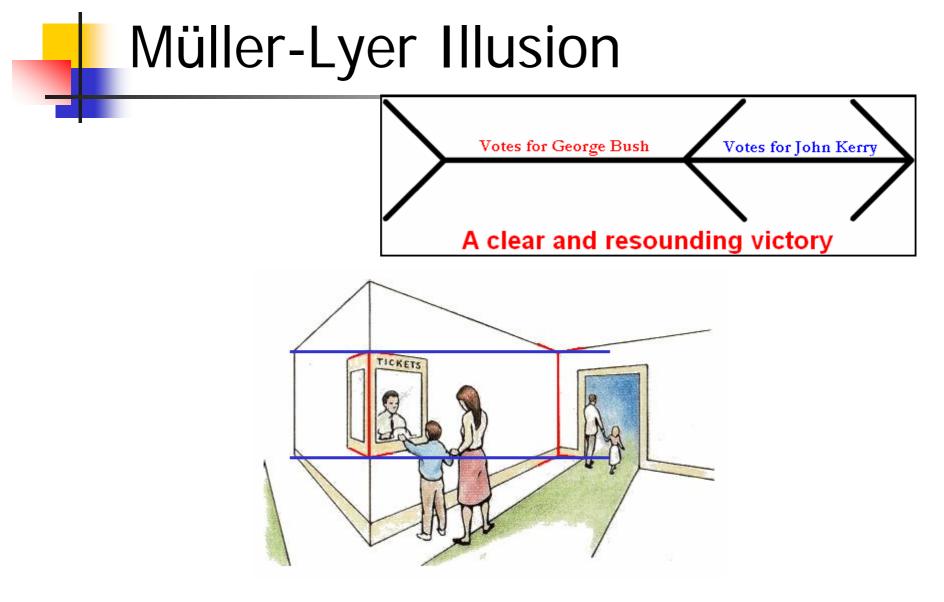
- Projective Geometry
- Homography



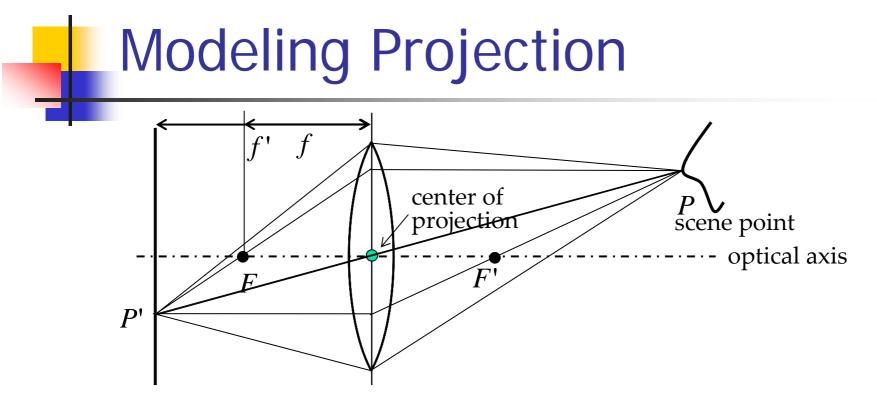
Ames Room

Readings

- Mundy, J.L. and Zisserman, A., Geometric Invariance in Computer Vision, Appendix: Projective Geometry for Machine Vision, MIT Press, Cambridge, MA, 1992
- available online: <u>http://www.cs.cmu.edu/~ph/869/papers/zisser-mundy.pdf</u>

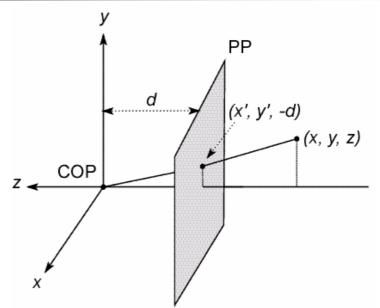


http://www.michaelbach.de/ot/sze_muelue/index.html



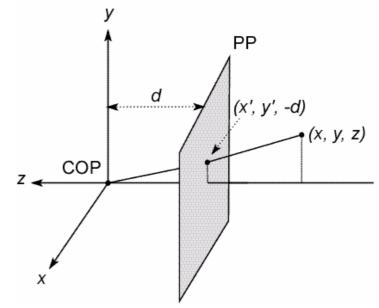
f': *effective* focal length (will be *d* from next slide)

Modeling Projection



- The coordinate system
 - We will use the pin-hole model as an approximation
 - Put the optical center (Center Of Projection) at the origin
 - Put the image plane (Projection Plane) in front of the COP
 - The camera looks down the *negative* z axis
 - we need this if we want right-handed-coordinates

Modeling Projection



Perspective Projection

- Compute intersection with PP of ray from (x,y,z) to COP
 Derived using similar triangles (x, y, z) → (-d^x/_z, -d^y/_z, -d)
- We get the projection by throwing out the last coordinate:

$$(x, y, z)
ightarrow (-drac{x}{z}, -drac{y}{z})$$

Homogeneous Coordinates

Is this a linear transformation?

- no-division by z is nonlinear $(x, y, z) \rightarrow (-d\frac{x}{z}, -d\frac{y}{z})$
- Trick: add one more coordinate:

$$(x,y) \Rightarrow \left[\begin{array}{c} x \\ y \\ 1 \end{array} \right]$$

 $(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

homogeneous image coordinates homogeneous scene coordinates
 Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, -d\frac{y}{z})$$

divide by third coordinate

- This is known as perspective projection
 - The matrix is the projection matrix

Perspective Projection

How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

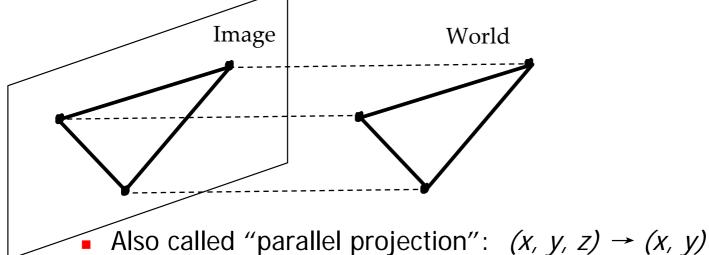
$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \end{bmatrix} \Rightarrow (-d\frac{x}{z}, -d\frac{y}{z})$$

Conclusion: the Projection matrix is scale independent

Orthographic Projection

Special case of perspective projection

Put effective optical center to infinite:



What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Other types of projection

- Scaled orthographic
 - Also called "weak perspective"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

- Affine projection
 - Also called "paraperspective"

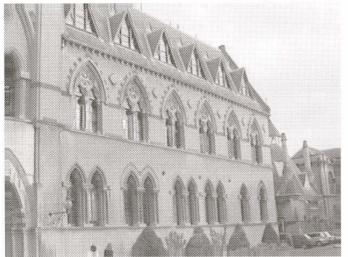
$$\left[\begin{array}{cccc}a&b&c&d\\e&f&g&h\\0&0&0&1\end{array}\right]\left[\begin{array}{c}x\\y\\z\\1\end{array}\right]$$

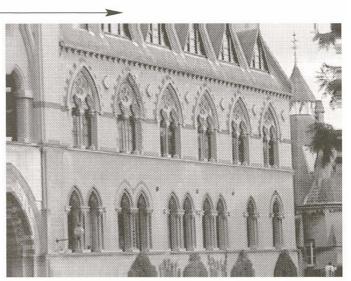
Weak-Perspective Projeciton

Scaled orthographic

increasing focal length

increasing distance from camera

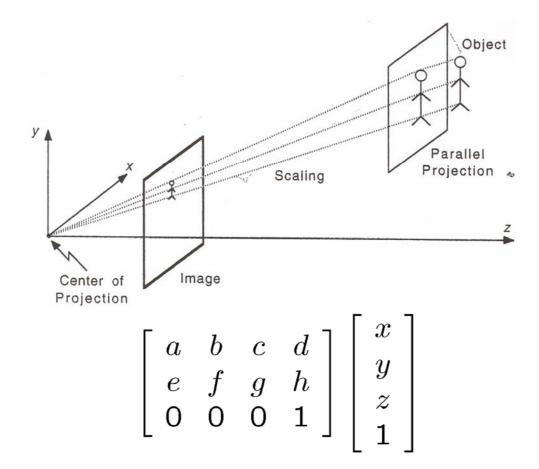




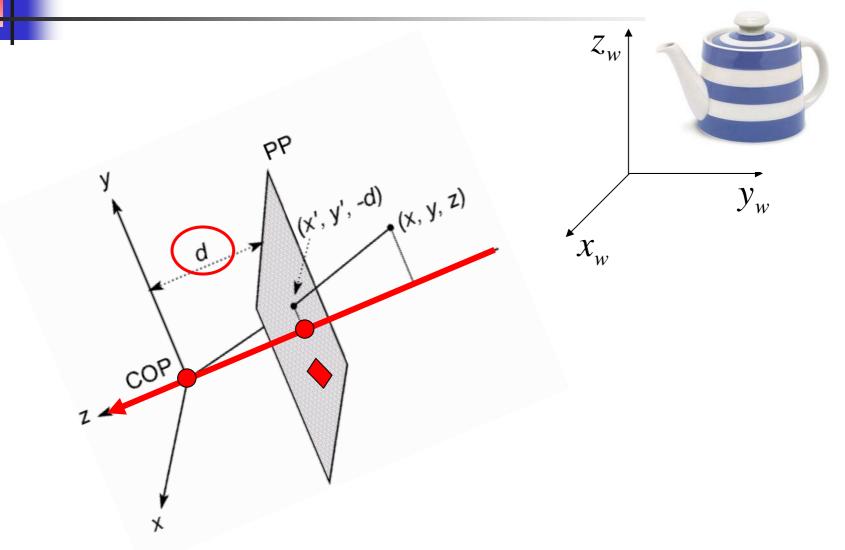
x $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$

Affine Projection

Also called "paraperspective"



Camera Parameters



Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

 \mathbf{x}

$$\mathbf{\Pi} = \begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{f}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}$$

intrinsics projection rotation translation

- The definitions of these parameters are **not** completely standardized
 - especially intrinsics-varies from one book to another

Camera Calibration

Goal: estimate the camera parameters

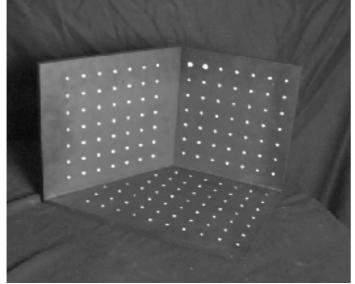
Version 1: solve for projection matrix

Version 2: solve for camera parameters separately

- intrinsics (focal length, principle point, pixel size)
- extrinsics (rotation angles, translation)
- radial distortion

Estimating the Projection Matrix

- Place a known object in the scene
 - identify correspondence between image and scene
 - compute mapping from scene to image



$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Issues

- must know geometry very accurately
- must know 3D->2D correspondence

Direct Linear Calibration

$$\begin{bmatrix} u_{i} \\ v_{i} \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_{i} \\ Y_{i} \\ Z_{i} \\ 1 \end{bmatrix}$$
$$u_{i} = \frac{m_{00}X_{i} + m_{01}Y_{i} + m_{02}Z_{i} + m_{03}}{m_{20}X_{i} + m_{21}Y_{i} + m_{22}Z_{i} + m_{23}}$$
$$v_{i} = \frac{m_{10}X_{i} + m_{11}Y_{i} + m_{12}Z_{i} + m_{13}}{m_{20}X_{i} + m_{21}Y_{i} + m_{22}Z_{i} + m_{23}}$$
$$u_{i}(m_{20}X_{i} + m_{21}Y_{i} + m_{22}Z_{i} + m_{23}) = m_{00}X_{i} + m_{01}Y_{i} + m_{02}Z_{i} + m_{03}$$
$$v_{i}(m_{20}X_{i} + m_{21}Y_{i} + m_{22}Z_{i} + m_{23}) = m_{10}X_{i} + m_{11}Y_{i} + m_{12}Z_{i} + m_{13}$$
$$\begin{bmatrix} X_{i} & Y_{i} & Z_{i} & 1 & 0 & 0 & 0 & -u_{i}X_{i} & -u_{i}Y_{i} & -u_{i}Z_{i} & -u_{i} \\ 0 & 0 & 0 & 0 & X_{i} & Y_{i} & Z_{i} & 1 & -v_{i}X_{i} & -v_{i}Y_{i} & -v_{i}Z_{i} & -v_{i} \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{22} \\ m_{13} \\ m_{20} \\ m_{21} \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

 $m_{22} m_{23}$

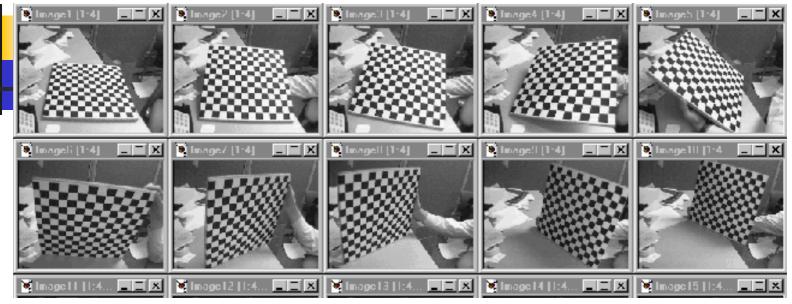
Direct linear calibration

- Advantage:
 - Very simple to formulate and solve
- Disadvantages:
 - Doesn't tell you the camera parameters
 - Doesn't model radial distortion
 - Hard to impose constraints (e.g., known focal length)
 - Doesn't minimize the right error function

For these reasons, *nonlinear methods* are preferred

- Define error function E between projected 3D points and image positions
 - E is nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques
 - e.g., variants of Newton's method (e.g., Levenberg Marquart)

Alternative: Multi-Plane Calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don't have to know positions/orientations
- Good code available online!
 - Intel's OpenCV library: <u>http://www.intel.com/research/mrl/research/opencv/</u>
 - Matlab version by Jean-Yves Bouget: <u>http://www.vision.caltech.edu/bouguetj/calib_doc/index.html</u>
 - Zhengyou Zhang's web site: <u>http://research.microsoft.com/~zhang/Calib/</u>

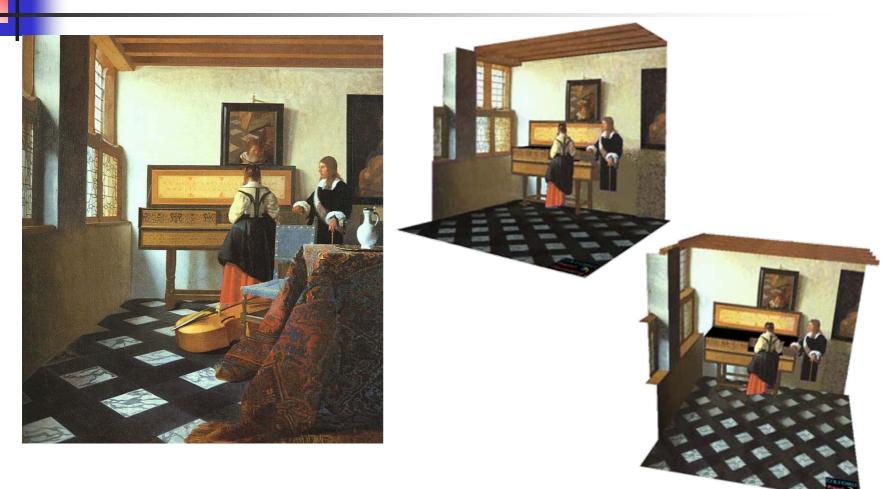
Chromaglyphs



Courtesy of Bruce Culbertson, HP Labs http://www.hpl.hp.com/personal/Bruce_Culbertson/ibr98/chromagl.htm Projective geometry —what's it good for?

- Uses of projective geometry
 - Drawing
 - Measurements
 - Mathematics for projection
 - Undistorting images
 - Focus of expansion
 - Camera pose estimation, match move
 - Object recognition

Applications of projective geometry



Reconstructions by Criminisi et al.

Measurements on planes

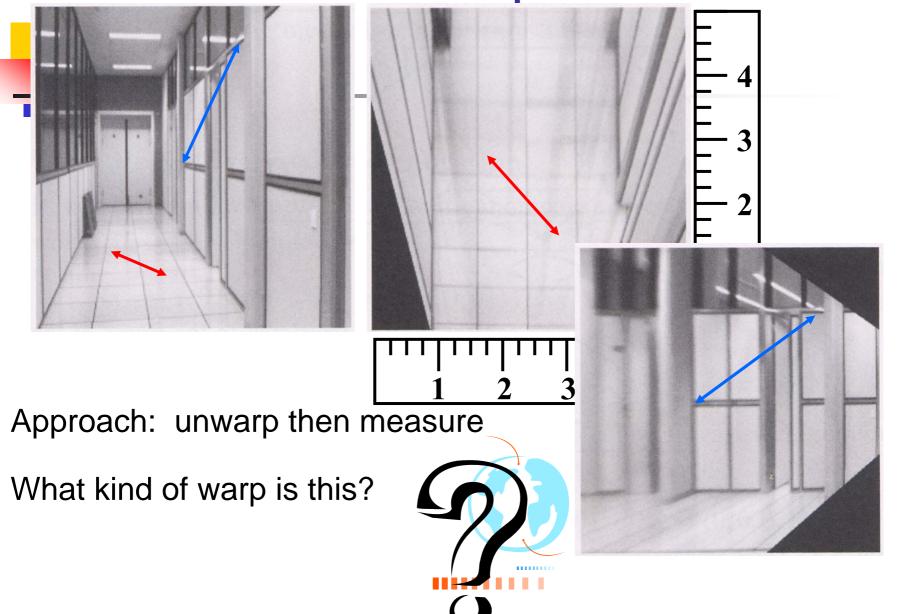
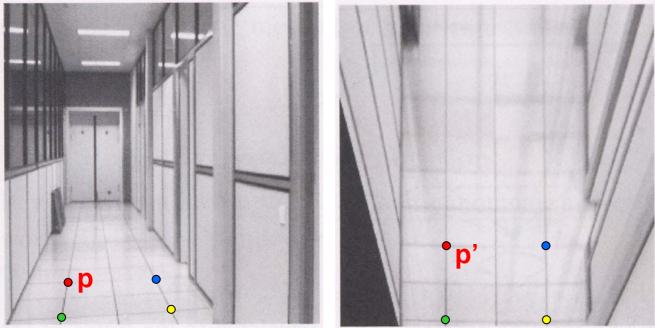


Image rectification



To unwarp (rectify) an image

- solve for homography H given p and p'
- solve equations of the form: wp' = Hp
 - linear in unknowns: w and coefficients of H
 - H is defined up to an arbitrary scale factor
 - how many points are necessary to solve for H?

Solving for homographies

$$\begin{bmatrix} x_i'\\y_i'\\1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02}\\h_{10} & h_{11} & h_{12}\\h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i\\y_i\\1 \end{bmatrix}$$

$$\begin{aligned} x'_{i} &= \frac{h_{00}x_{i} + h_{01}y_{i} + h_{02}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}} \\ y'_{i} &= \frac{h_{10}x_{i} + h_{11}y_{i} + h_{12}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}} \\ x'_{i}(h_{20}x_{i} + h_{21}y_{i} + h_{22}) &= h_{00}x_{i} + h_{01}y_{i} + h_{02} \\ y'_{i}(h_{20}x_{i} + h_{21}y_{i} + h_{22}) &= h_{10}x_{i} + h_{11}y_{i} + h_{12} \\ x_{i} & y_{i} & 1 & 0 & 0 & 0 & -x'_{i}x_{i} & -x'_{i}y_{i} & -x'_{i} \\ 0 & 0 & 0 & x_{i} & y_{i} & 1 & -y'_{i}x_{i} & -y'_{i}y_{i} & -y'_{i} \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

 h_{22}

Solving for homographies

Defines a least squares problem: minimize $\|Ah - 0\|^2$

- Since \boldsymbol{h} is only defined up to scale, solve for unit vector $\boldsymbol{\hat{h}}$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^{\mathsf{T}}\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

3D to 2D: "perspective" projection

Matrix Projection:

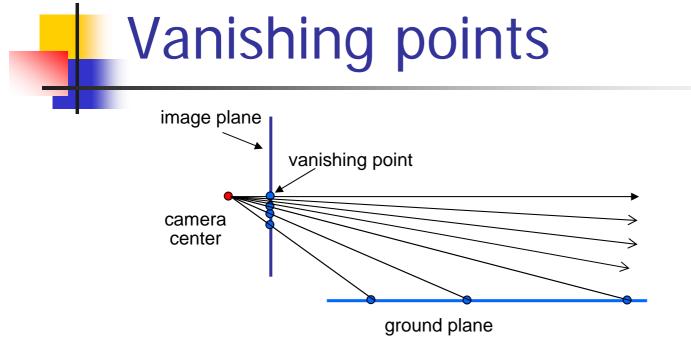
- What is *not* preserved under perspective projection?
- What IS preserved?

Homographies of points and lines

- Computed by 3x3 matrix multiplication
 - To transform a point: p' = Hp
 - To transform a line: $lp=0 \rightarrow l'p'=0$
 - $0 = Ip = IH^{-1}Hp = IH^{-1}p' \Rightarrow I' = IH^{-1}$
 - Ines are transformed by postmultiplication of H⁻¹

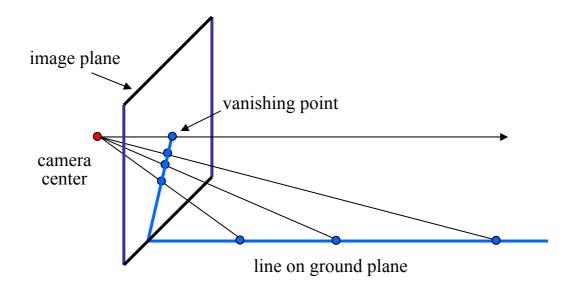
3D projective geometry

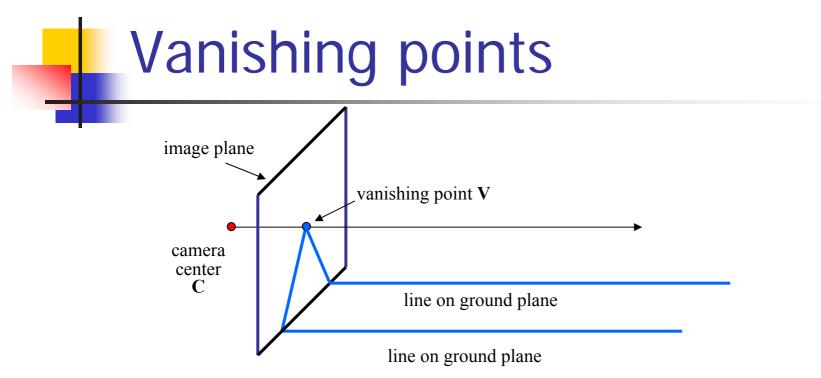
- These concepts generalize naturally to 3D
 - Homogeneous coordinates
 - Projective 3D points have four coords: $\mathbf{P} = (X, Y, Z, W)$
 - Duality
 - A plane N is also represented by a 4-vector
 - Points and planes are dual in 3D: N P=0
 - Projective transformations
 - Represented by 4x4 matrices T: P' = TP, $N' = N T^{-1}$



Vanishing pointprojection of a point at infinity

Vanishing points (2D)





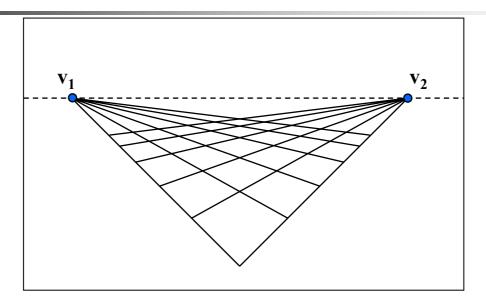
- Properties
 - Any two parallel lines have the same vanishing point v
 - The ray from **C** through **v** is parallel to the lines
 - An image may have more than one vanishing point
 - in fact every pixel is a potential vanishing point

Vanishing points



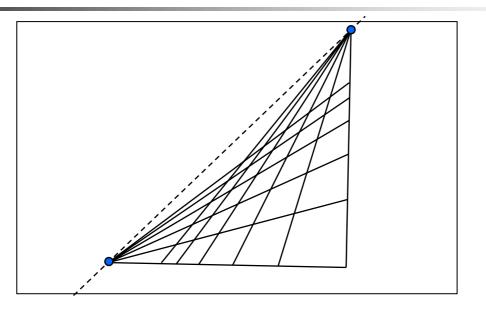
Image by Q-T. Luong (a vision researcher & photographer)

Vanishing lines

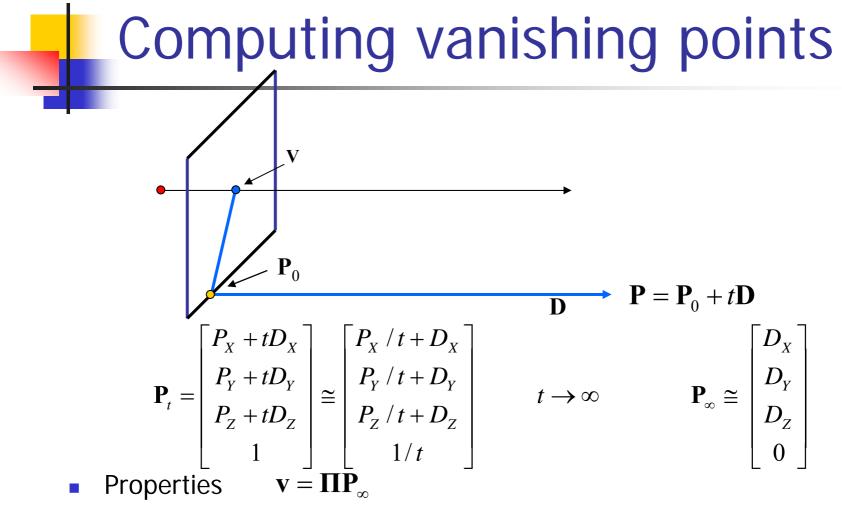


- Multiple Vanishing Points
 - Any set of parallel lines on the plane define a vanishing point
 - The union of all of these vanishing points is the *horizon line*
 - also called vanishing line
 - Note that *different* planes define *different* vanishing lines

Vanishing lines

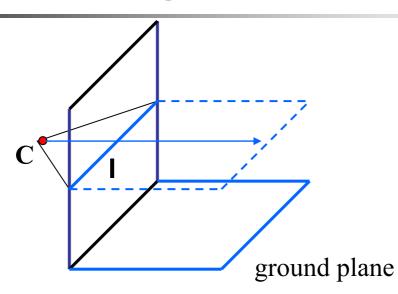


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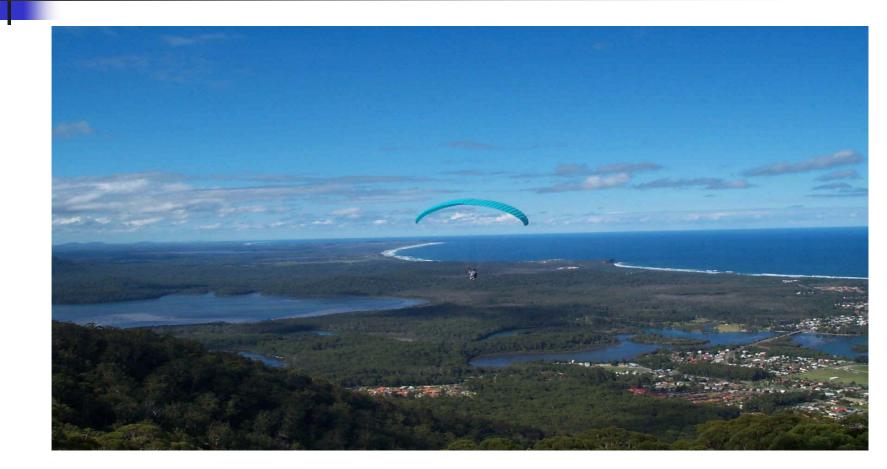


- P_∞ is a point at *infinity*, v is its projection
- They depend only on line *direction*
- Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at \mathbf{P}_{∞}

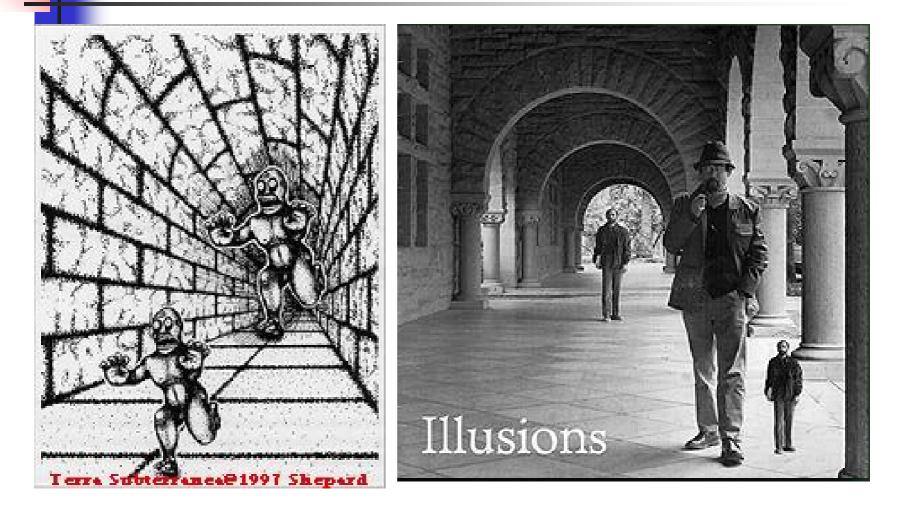
Computing vanishing lines

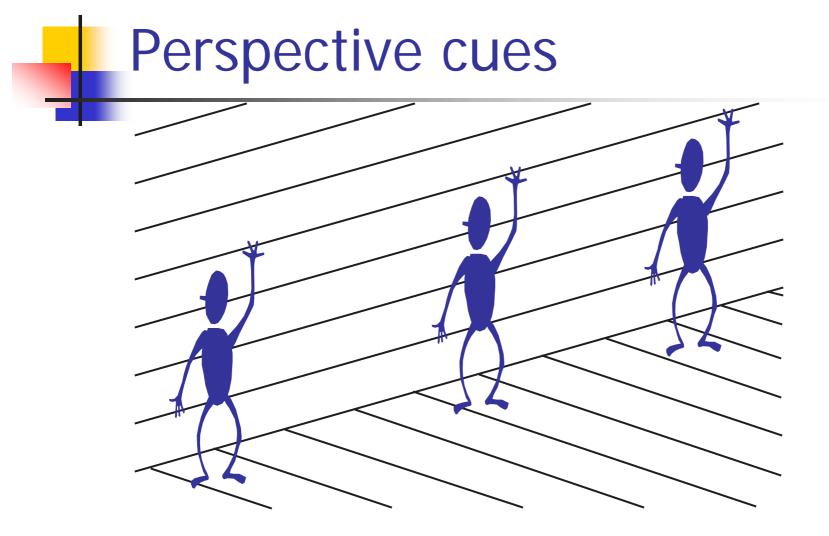


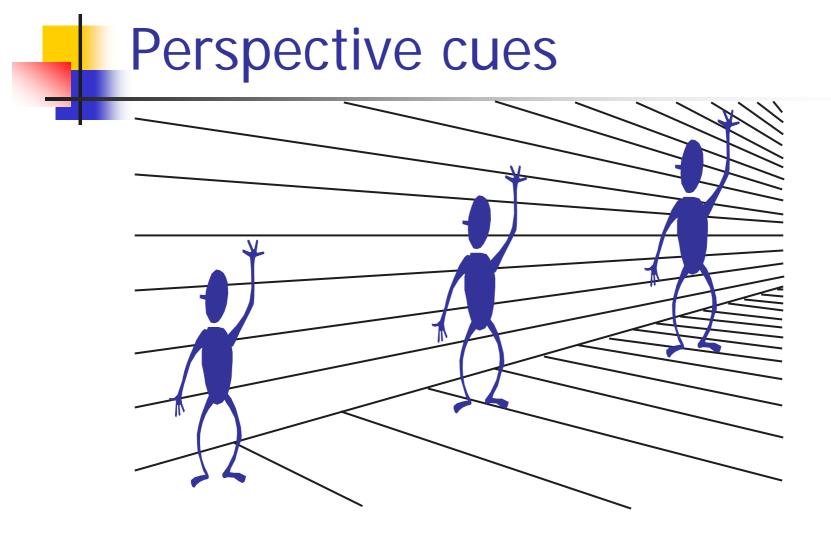
- Properties
 - I is intersection of horizontal plane through C with image plane
 - Compute I from two sets of parallel lines on ground plane
 - All points at same height as C project to I
 - points higher than C project above I
 - Provides way of comparing height of objects in the scene

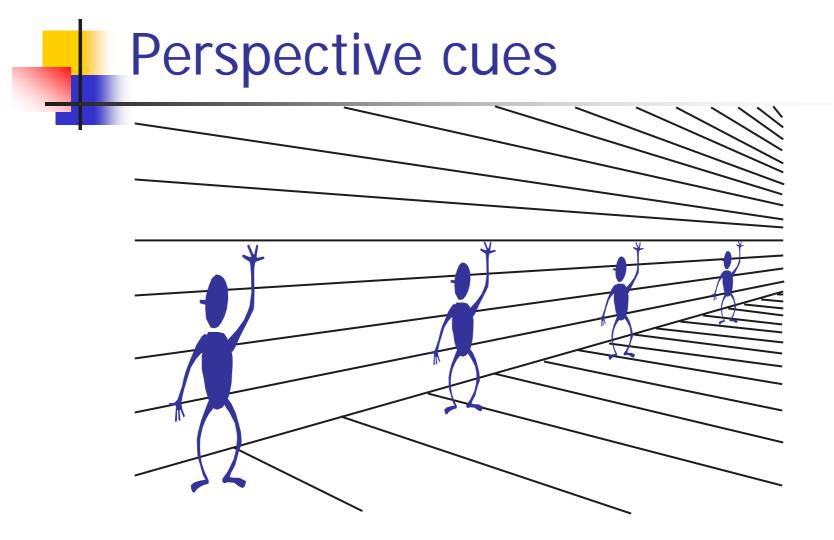


Fun with vanishing points

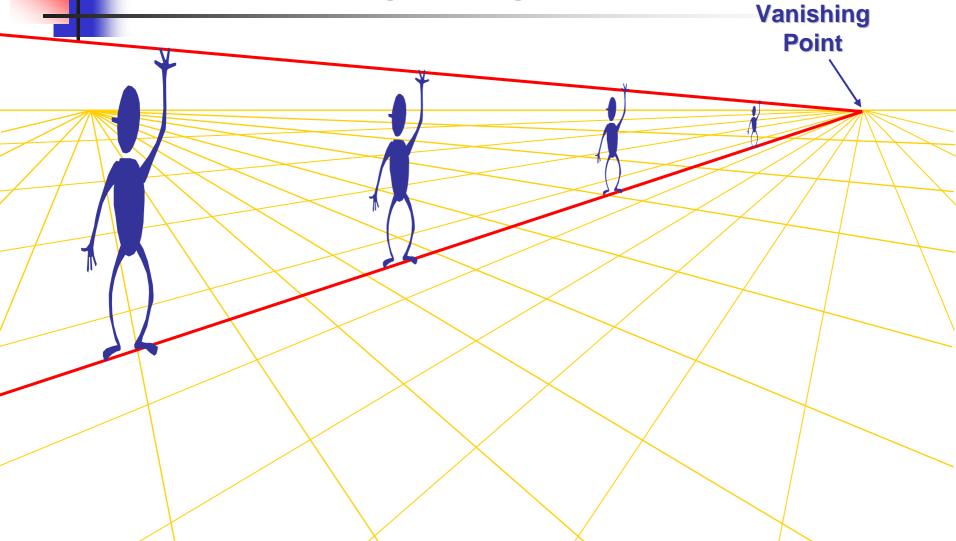


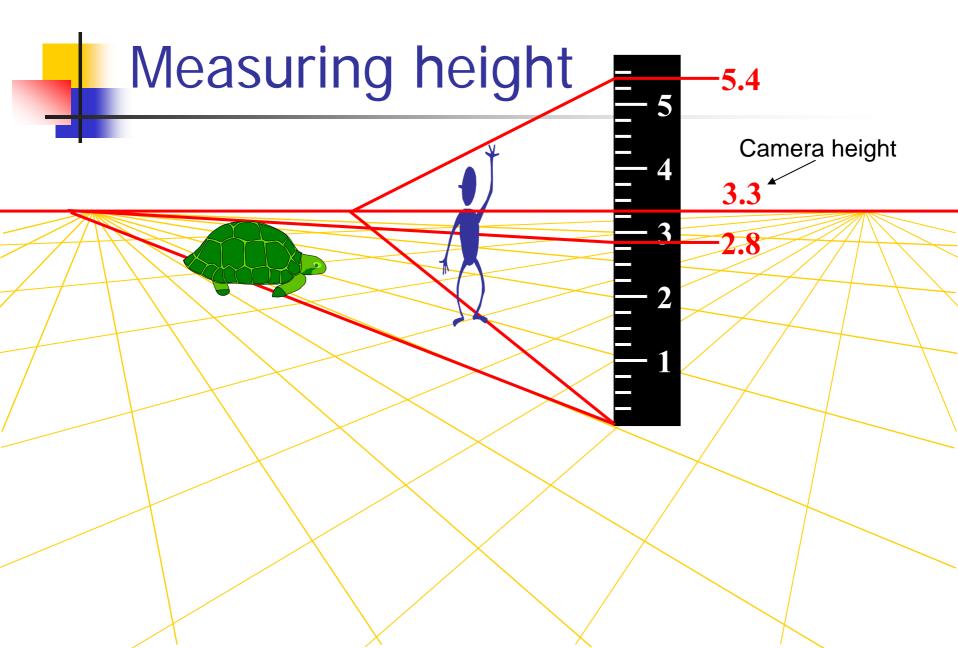


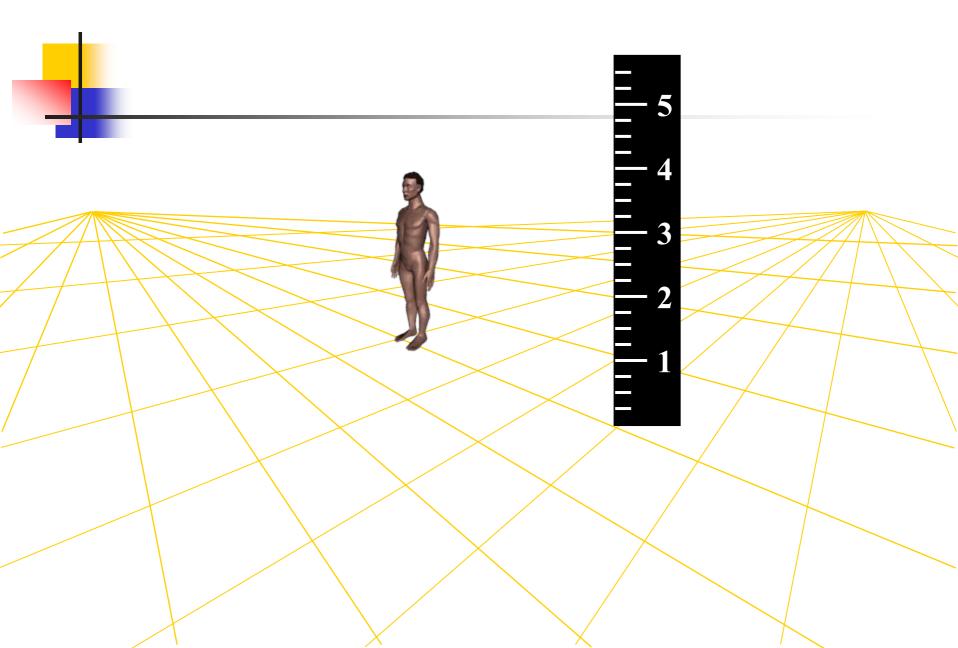




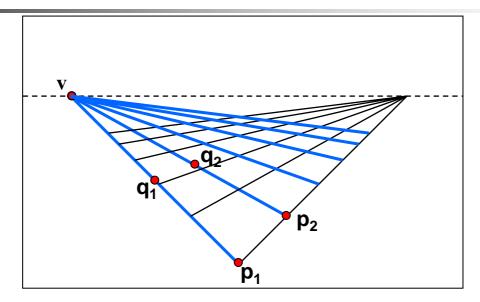
Comparing heights





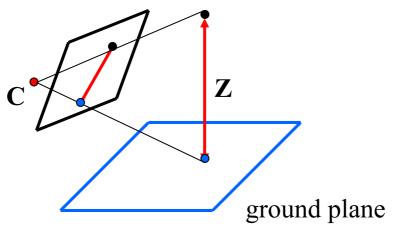


Computing vanishing points (from lines)



- Intersect p_1q_1 with p_2q_2 $v = (p_1 \times q_1) \times (p_2 \times q_2)$
- Least squares version
 - Better to use more than two lines and compute the "closest" point of intersection
 - See notes by <u>Bob Collins</u> for one good way of doing this:
 - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Measuring height without a ruler



Compute Z from image measurements Need more than vanishing points to do this

The cross-ratio of 4 collinear points

The cross ratio

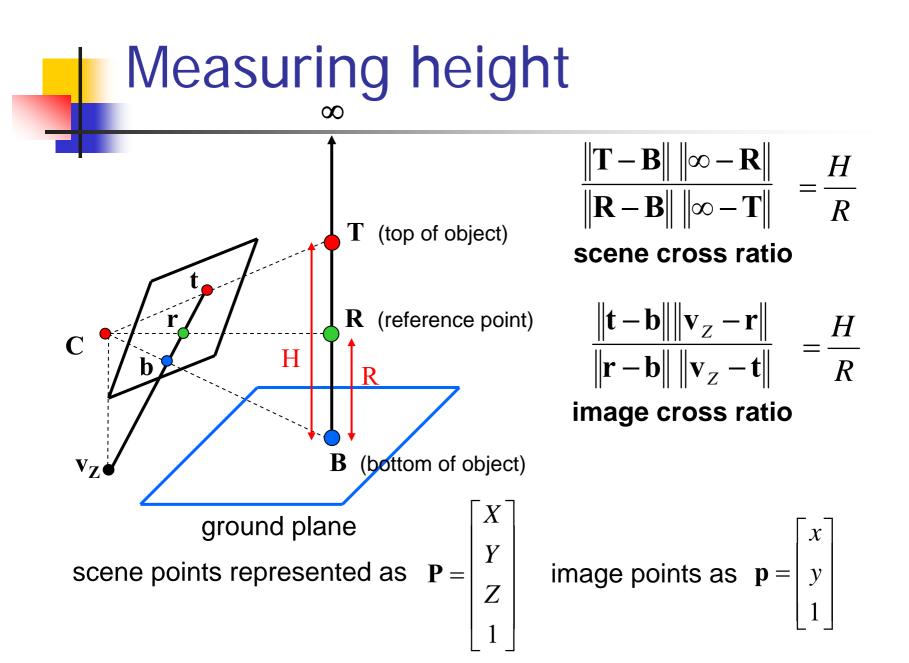
- A Projective Invariant
 - Something that does not change under projective transformations (including perspective projection)

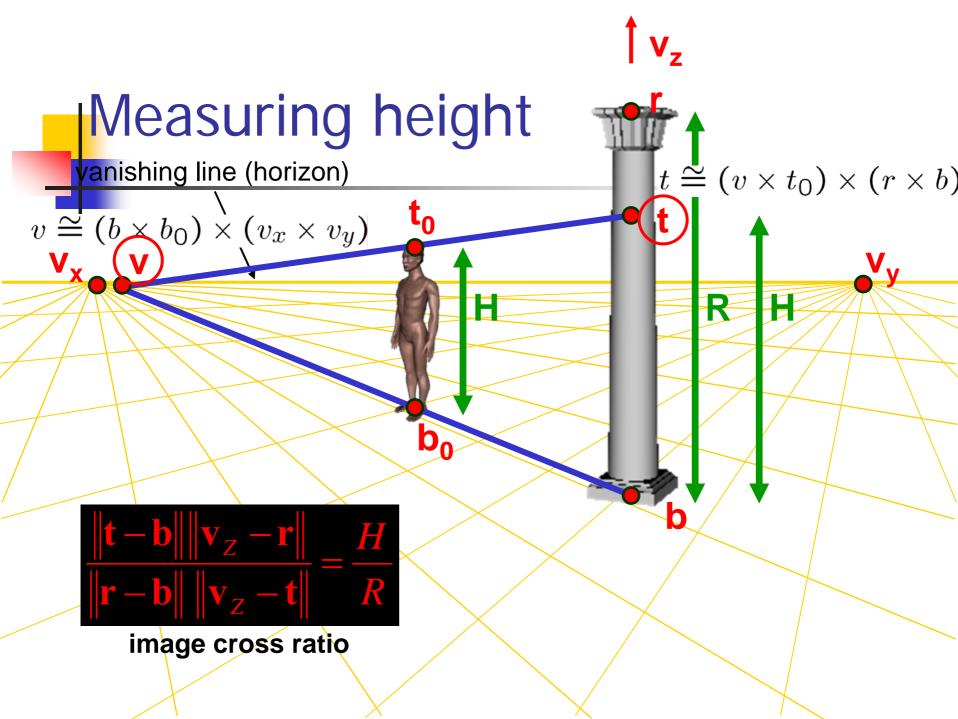
 $\mathbf{P}_i = \begin{vmatrix} \mathbf{X}_i \\ \mathbf{Y}_i \\ \mathbf{Z}_i \end{vmatrix}$

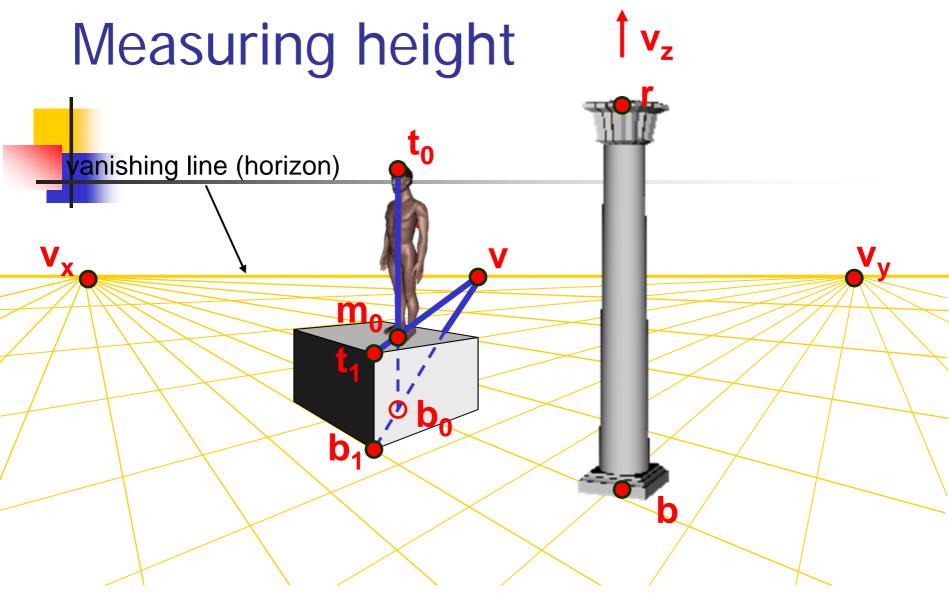
Can permute the point ordering

 $\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \|\mathbf{P}_4 - \mathbf{P}_3\|}$ 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry







What if the point on the ground plane b_o is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find b_o as shown above

Computing (X,Y,Z) coordinates

- Okay, we know how to compute height (Z coords)
- how can we compute X, Y?



Easy – Just map it to the reference plane

Vanishing point calibration

- Advantages:
 - only need to see vanishing points (e.g., architecture, table, ...)
- Disadvantages:
 - not that accurate
 - need rectahedral object(s) in scene

Single View Metrology

- A. Criminisi, I. Reid and A. Zisserman (ICCV 99)
- Make scene measurements from a single image
 - Application: 3D from a single image
- Assumptions
 - 1 3 orthogonal sets of parallel lines
 - ² 4 known points on ground plane
 - ³ 1 height in the scene
- Can still get affine reconstruction without 2 and 3

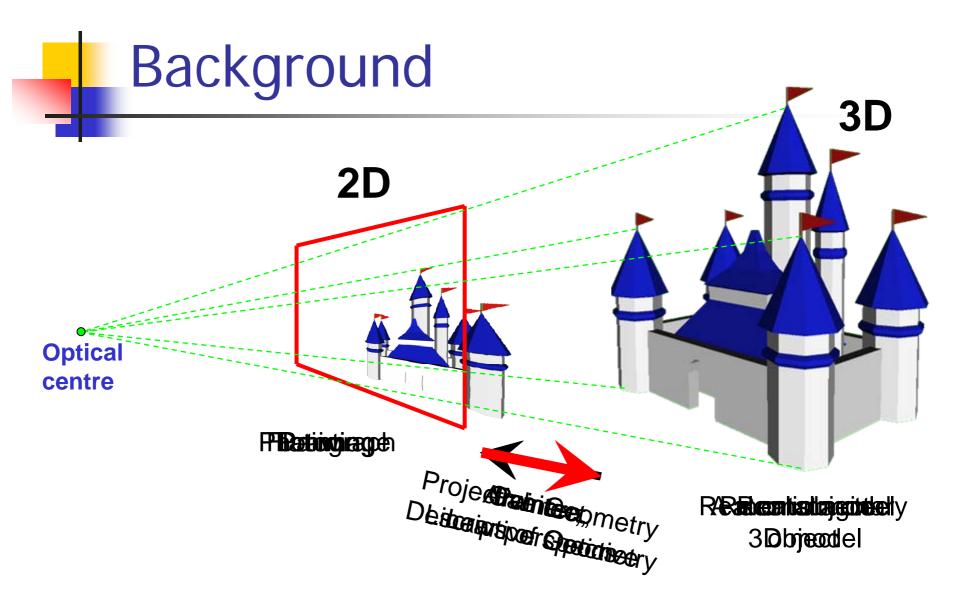
Problem

How?

Why?

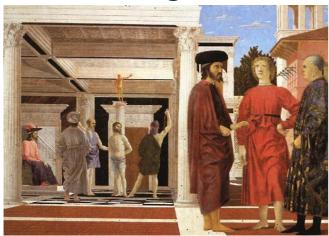
Is it possible to extract 3D geometric information from single images?





Introduction

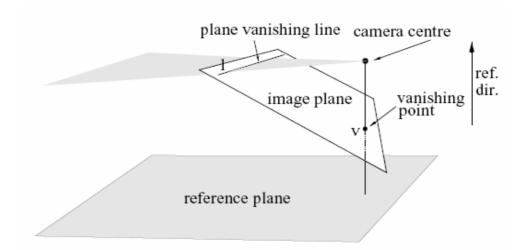
3D affine measurements may be measured from a single perspective image





Geometry

- Overview
- Measurements between parallel lines
- Measurements on parallel planes
- Determining the camera position



Assumptions

- Assume that images are obtained by perspective projection
- Assume that, from the image, a:
 - vanishing line of a reference plane
 - vanishing point of another reference direction

may be determined from the image

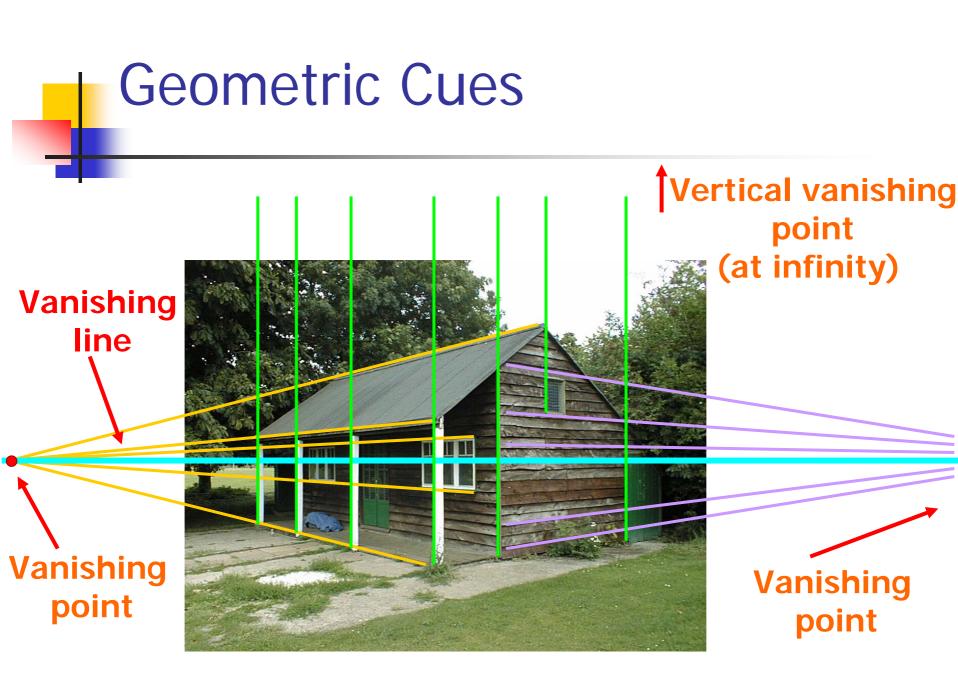
Geometric Cues

- Vanishing Line ℓ
 - Projection of the line at infinity of the reference plane into the image

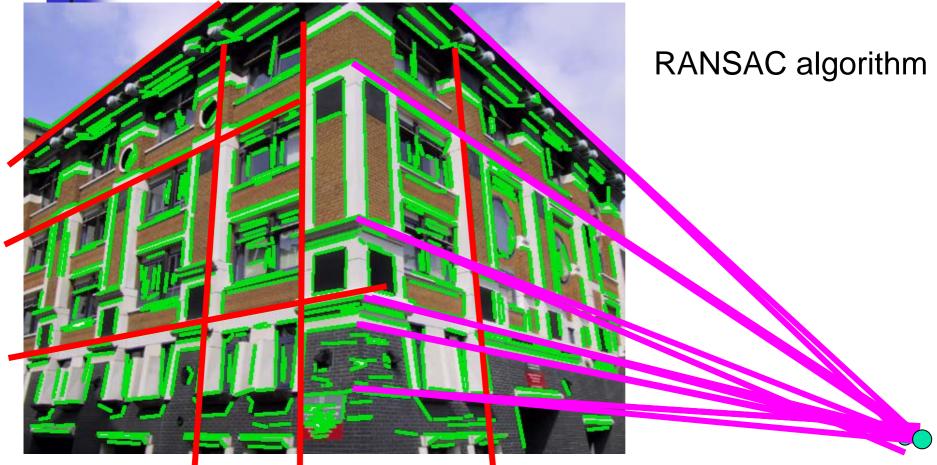


Geometric Cues

- Vanishing Point(s) v
 - A point at infinity in the reference direction
 - Reference direction is NOT parallel to reference plane
 - Also known as the vertical vanishing point



Automatic estimation of vanishing points and lines



Candidate vanishing point

Automatic estimation of vanishing points and lines



a

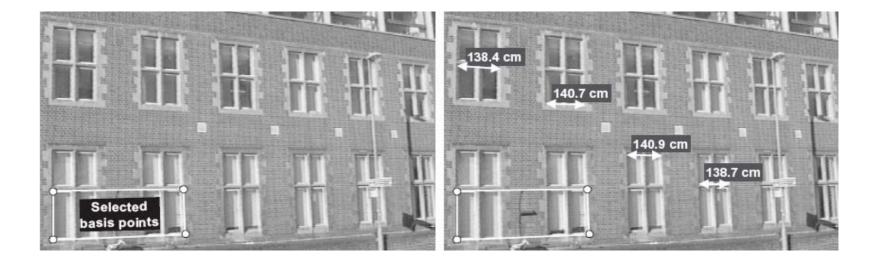


Estimating Height



- •The distance $|| t_r b_r ||$ is known
- •Used to estimate the height of the man in the scene

Parallel Line Segments

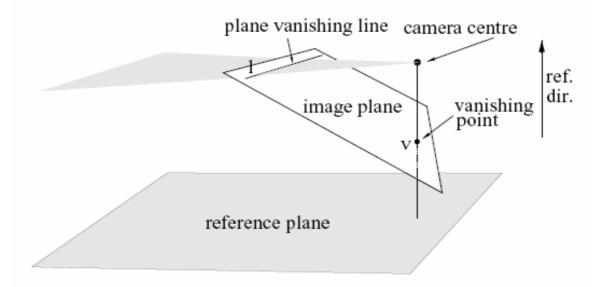


 Basis points are manually selected and measured in the real world

•Using ratios of lengths, the size of the windows are calculated

Camera Position

- Using the techniques we developed in the previous sections, we can:
 - Determine the distance of the camera from the scene
 - Determine the height of the camera relative to the reference plane

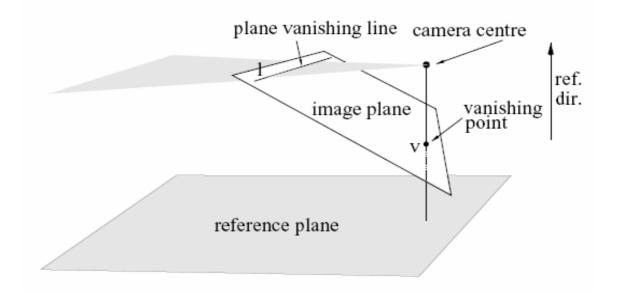


Camera Distance from Scene

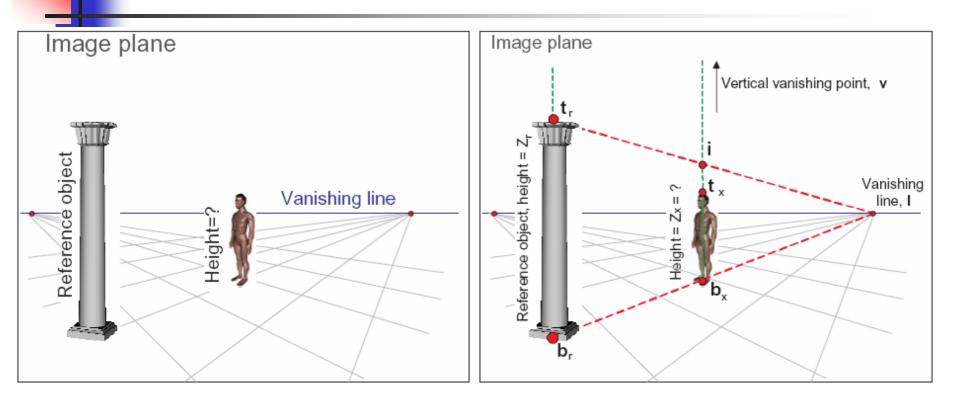
- In Measurements between Parallel Lines, distances between planes are computed as a ratio relative to the camera's distance from the reference plane
- Thus we can compute the camera's distance from a particular frame knowing a single reference distance

Camera Position Relative to Reference Plane

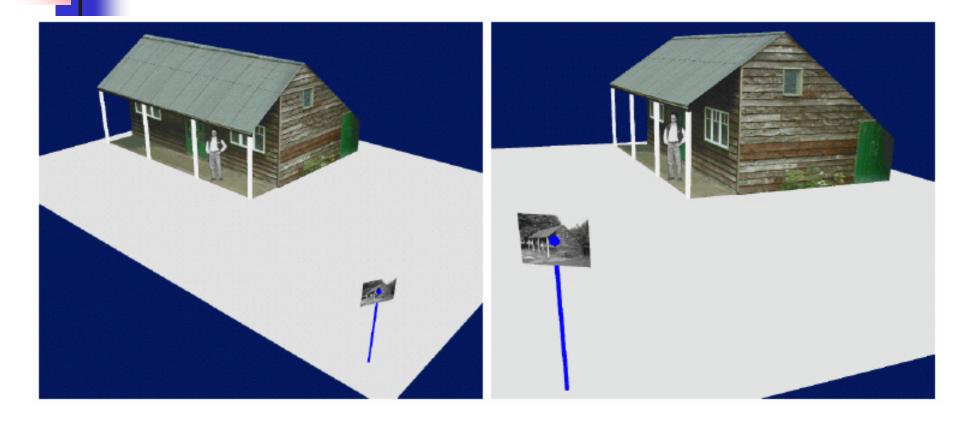
The location of the camera relative to the reference plane is the back-projection of the vanishing point onto the reference plane



Representation



Camera In Scene



Applications

- Forensic Science (法医科学)
 - Height of suspect
- Virtual Modeling
 - 3D reconstruction of a scene
- Art History
 - Modeling paintings

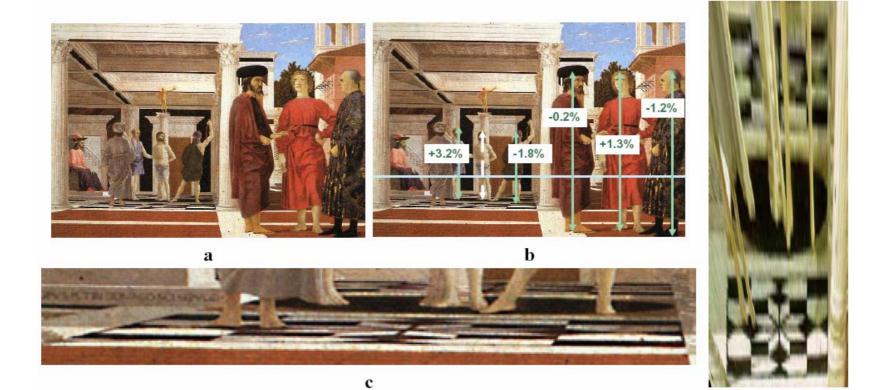
Forensic Science



Virtual Modeling



Art History



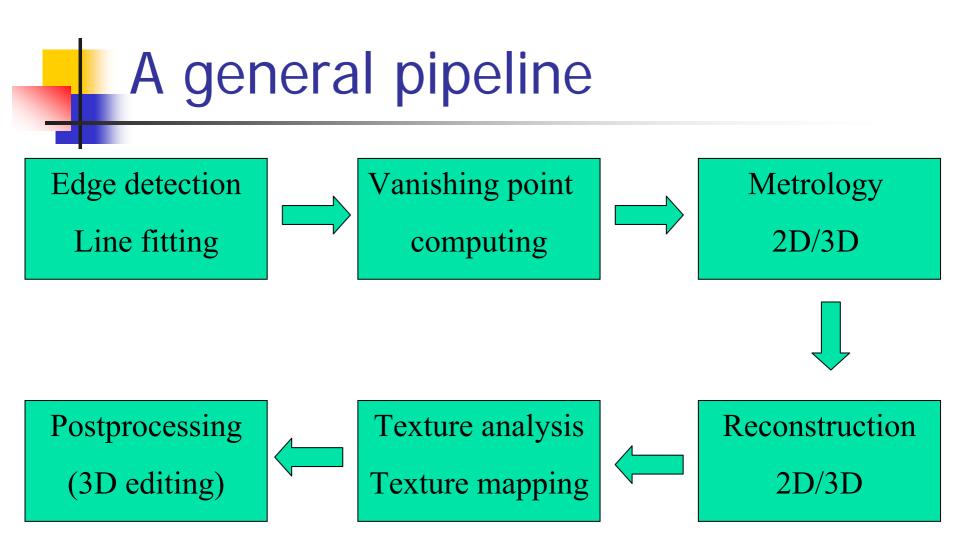
Conclusions

- Affine structure of 3D space may be partially recovered from perspective images
- Measurements between and on parallel planes can be determined
- Practical applications can be derived

Criminisi et al., ICCV 99

Complete approach

- Load in an image
- Click on parallel lines defining X, Y, and Z directions
- Compute vanishing points
- Specify points on reference plane, ref. height
- Compute 3D positions of several points
- Create a 3D model from these points
- Extract texture maps
- Output a VRML model



SingleView/Detection/fitting/calibration/metrology/optimization/ Structure/modeling/Texture/editing/...

3D Modeling from a Photograph



3D Modeling from a Photograph



Assignment/Project

- Any work related to image and vision computing is acceptable
- Presentation at the end of the semester
 - Pre-submission of demos, codes, and documents
 - PPT and DEMO at presentation
 - Each student has around 30 minutes

Good work win exemption of final exam

Assignment/Project presentation

- June 3rd, Tuesday, 16:50-18:40
- Presentation: powerpoint slides, 30 minutes
- Presubmission (By June 1st, 23:59):
 - Codes (concise) with comments (Detailed)
 - Distinct your work from open sources (if any)
 - Result demos (Mandatory)
 - Documents (Emphasis on your own work)
- The sooner, the better.

Document/PPT format

- Abstract(摘要)
- Introduction(引言)
- Related work(相关工作)
- Your work--major part(你的工作——重点)
 - Main idea(思想)
 - Global framework / Step-by-step pipeline (整体框架)
 - Implementation details / Algorithms(实现细节)
- Experimental result(试验结果)
- Conclusion and future work(结论)
- Please refer to formal technical papers

Source code format

Executable

Comment whenever possible (with your name)

- Open source: highlight your understanding
- Your work: tell technical detail
- Sample:
 - glCallList(iframe % Npat + 1); // chenyisong:每Npat=32帧一循环,每一帧调用一个显示列表
 - glBegin(GL_QUAD_STRIP); // chenyisong:将新的背景噪音作为源混合,为什么这里只绘制一个单位方格?

Make your work understandable and convincible